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# PROBLEMS CONCERNING THE STABILITY AND MANEUVERABILITY OF AIRPLANES

By Jean Biche

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# PROBLEMS CONCERNING THE STABILITY AND MANEUVERABILITY OF AIRPLANES*

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The stability of an airplane can be easily determined by wind-tunnel tests, especially by simple tests with models mounted wind-vane fashion. However, each stability curve plotted by this method is valid only for a certain setting of the corresponding control surface, i.e. it characterizes the stability of the airplane with the control stick locked in a given position. We may then inquire as to what the stability conditions are when the controls are released, or (the control stick being locked) when the particular control rods described in this report leave a certain degree of freedom to the control surface. In this particular case we shall study the effect of the system on the maneuverability and at the same time seek to determine the relation between the displacements of the controls and the deflections of the control surfaces insuring better maneuverability.

The problems thus defined, are studied below from the point of view of longitudinal and transverse stability. Directional stability is not included in this study, as it is analogous to longitudinal stability, while the complete study of the effect of rudder deflections on the airplane would exceed the limits of the present investigation.

## I. STABILITY AND MANEUVERABILITY ABOUT THE LATERAL AXIS Y

### a) Stability of an Airplane Having a Stabilizer and an Elevator, when the Control of the Latter is Released

The friction of the control is first assumed to be negligible, its weight and that of the elevator being zero

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* "Etude de quelques Problèmes concernant la stabilité et la maniabilité des avions." Revue de la Société Générale Aéronautique, January, 1932, pp. 12-19.

(static equilibrium and no inertia). Under these conditions the elevator always assumes the angle of zero hinge moment. The stability can then be investigated in the wind tunnel, after demonstrating the following proposition:

"When the elevator is free to move about its hinge, everything takes place as if the area S of the horizontal empennage had become S', which has a constant value regardless of speed and incidence."

According to Toussaint ("L'Aviation Actuelle"), the unit hinge moment is given by the expression

$$C_{m_c} = ni_e + p\beta$$

where $i_e$ is the incidence of the stabilizer (fig. 1),

$\beta$ the elevator setting, and

$n$ and $p$ constant quantities.

Hence, in the case considered,

$$ni_e + p\beta = 0$$

$$\beta = -\frac{n}{p} i_e \qquad (1)$$

The incidence of the direction of zero lift is $i_e + m\beta$, $m$ being a coefficient nearly constant within the ordinary range of the angles $\beta$. The lift of the horizontal empennage is therefore

$$F = \frac{C_z}{16} S V^2 = \frac{A\ (i_e + m\beta)}{16} SV^2 \qquad (2)$$

On replacing $\beta$ by its value (1), we have

$$F = \frac{Ai_e}{16}\left(1 - \frac{mn}{p}\right) SV^2 \qquad (3)$$

On the other hand, for an indeformable empennage with an area S', we have

$$F = \frac{Ai_e}{16} S'V^2 \qquad (4)$$

On equalizing 3 and 4, we obtain

$$S\left(1 - \frac{mn}{p}\right) = S'$$

or

$$\frac{S'}{S} = 1 - \frac{mn}{p}$$

and, on replacing m and n by their values (still according to Toussaint), we get

$$\boxed{\frac{S'}{S} = 1 - \frac{0.25\,A\,\sigma}{0.25\,A - 0.4 + 0.4\,\sigma}} \qquad (5)$$

A is defined by $d(100\ C_z) = A\ di_e$. Equation 5 shows that $S'$ is smaller than S. Hence, at any incidence, the pilot reduces the stability by releasing the control, $S'/S$ being independent of the incidence. It is likewise seen that $S'/S$ is always smaller than $1 - \sigma$. The theoretical area $S'$ is therefore smaller than the area of the stabilizer. The curve $Cmg = f(i)$ of the unit moment about the c.g. may then be plotted as a function of the airplane incidence, by beginning with the Cmg curve corresponding to the two cases in one of which the elevator is in line with the stabilizer and in the other of which the airplane is deprived of its horizontal empennage, both curves being obtained by wind-vane tests.

The unit moment of an airplane, having a horizontal empennage with an area $S'$, is deduced from the moments obtained for the areas S and O by a simple interpolation

$$\boxed{Cmg_{S'} = Cmg_{S=0} + \frac{S'}{S}\left(Cmg_S - Cmg_{S=0}\right)} \qquad (6)$$

This expression is justified as follows. The tail surface S develops a unit moment $\Delta Cmg_S$ about the c.g.

$$Cmg_S = Cmg_{S=0} + \Delta Cmg_S \qquad (7)$$

while the tail surface $S'$ produces a unit moment

$$\Delta\, Cmg_{S'} = \frac{S'}{S}\ \Delta\ Cmg_S$$

$$Cmg_{S'} = Cmg_{S=0} + \frac{S'}{S}\ \Delta\ Cmg_S \qquad (8)$$

By eliminating $Cmg_s$ from equations 7 and 8, we obtain

$$Cmg_{s'} - \frac{S'}{S} Cmg_s = Cmg_o \left(1 - \frac{S'}{S}\right)$$ from which

equation 6 is derived. This method, developed by Mr. Rivière of the Hanriot branch of the "Societé Générale Aéronautique" has the following consequences. The desired curve $Cmg = f(i)$ is known and may belong to one of the three following types:

Cases I and II.- When the control stick is released, the airplane always establishes equilibrium at the incidence $i_o$ (stable equilibrium) even when the curve (case II) has negative slopes.

Case III.- If the stick is released at an incidence $i < i_2$, the angle of equilibrium is $i_1$. If $i > i_2$, the angle of equilibrium is $i_o$. This case may be dangerous.

It is interesting to note that the airplane may become unstable (II) and still return automatically to an incidence of normal flight regardless of the incidence at which the controls are released. Safety in flight is always assured in cases I and II, and also in case III, provided $i_2$ is smaller than the incidence of zero lift corresponding to a vertical dive.

If the elevator control is not statically balanced, the elevator is depressed by its own weight, thus tending to bring the airplane into a dive. The incidence of equilibrium $i_o$ is thus shifted toward the left (reduced) at a rate inversely proportional to the speed, since the aerodynamic forces, which vary as the square of the speed, increase greatly as compared with the weight of the elevator. Since, on the other hand, the gliding speed increases when $i_o$ decreases, the incidence of equilibrium is never excessively reduced, even when the controls are not balanced.

The only dangerous modification of the above flight conditions is caused by heavy control wheels (often used on seaplanes), which introduce a diving stress directly proportional to the diving angle of the airplane. Such wheels are designed to withstand the maximum stress applied by the pilot. They have practically the same weight on all airplane types, and their action, prejudicial to longitudinal stability with released controls, is inversely proportional to the size and speed of the airplane.

Lastly, if the control produces friction and if the inertia of the elevator is appreciable, the latter does not readily respond to the action of the aerodynamic forces. The tail-surface area must therefore be S", intermediate between S', as calculated above, and S, thus improving the stability.

b) Stability of an Airplane without Stabilizer

This is a special case of the above problem. The hinge axis of the horizontal empennage, consisting of an elevator only, is necessarily located in the focus (at 25 per cent of the mean chord), or forward of the focus. Two cases must be considered.

1. The tail surface is movable about an axis located at 25 per cent of the chord. The hinge moment is zero and the released control surface does not change its setting. The control stick may be held or released without changing the degree of stability.

2. The axis of the elevator hinge is located forward of the focus. Inasmuch as the elevator then assumes the incidence of zero moment, its lift is always zero (or constant, if the section is dissymmetrical), and its efficiency is also zero. This case is the same as that in which the horizontal empennage is entirely removed. In general the result is a very marked instability. The effect of friction, of the weight of the control surface and of its inertia is the same as in the above-treated general case.

c) Case of Special Control Rods

Special control systems have been evolved for making the "stick-elevator" connection conform to a given law. This applies to the system known as the "variable-sensitivity" controls, in which the relation between the angular deflections of the control stick and of the elevator varies either as the setting, as the speed, or as both factors simultaneously. Such systems affect the maneuverability, as well as the stability of airplanes. We will first describe this type of control, and then consider its effect on the characteristics of an airplane.

1. Control with sensitivity variable according to the setting of the control surface.- A control has variable sensitivity when, for a shift $\Delta\alpha$ of the stick, the elevator setting changes by $\Delta\beta$, the ratio $\Delta\beta/\Delta\alpha$ varying with $\alpha$. In other words, if the diagram of the $\beta$ values is plotted against the values of $\alpha$, the resulting curve is not a straight line. If the curve $\Delta\beta$ is similar to that of Figure 3, where $\Delta\beta/\Delta\alpha$ increases with $\alpha$, the following advantages are realized.

Slight displacements of the stick to either side of the neutral position cause small elevator deflections. This prevents slight motions of the pilot's hand, voluntary or not, from causing irregular undulations of the airplane, especially if it is of a very fast and maneuverable type. At large values of $\alpha$ an angle $\Delta\alpha$ corresponds to a very large $\Delta\beta$. The effort required of the pilot is thus considerably increased, thus preventing abrupt maneuvers, at least at high speeds, since the maximum deflection of the control surface can not be attained.

On the other hand, at low speeds, the pilot can utilize the full stroke of the control stick and give the elevator its maximum deflection. Such a law of variation of the elevator deflections can be applied by various mechanical devices. The three-rod system affords a simple solution of the problem. The control stick DCBB' is pin-jointed at B and B' to the rods AB and A'B', hinged at A and A', which are integral with the airplane. The elevator control is connected at C.

2. Controls with sensitivity variable according to setting and speed.- A device of this type, which we consider of particular interest, is made as follows. An elastic unit is inserted between the control stick and the elevator. The Young's modulus of this system increases gradually with the stress or, in other words, a given stress variation causes a deformation of the system inversely proportional to the stress F, this condition being produced by a piston compressing air into a cylinder. $p_0$ being the initial air pressure and $\omega$ the piston section, the stress exerted to the right of the piston is (fig. 5)

$$F_2 = \rho_o \frac{d}{d + 2x} \omega$$

$$F = F_1 - F_2 = \rho_o \omega d \left( \frac{1}{d - 2x} - \frac{1}{d + 2x} \right)$$

whence
$$F = \frac{4 \rho_o \omega d x}{d^2 - 4x^2}$$

the variation of $F$ being shown in Figure 6.

## The Effect of Such a System on the Maneuverability

In the first place this system embodies the above-mentioned advantages of variable sensitivity. The curve $\alpha\beta$ is similar to that of Figure 3. When the control stick is shifted from its neutral position, the pressure of the air on the piston is very slight, and the elevator deflection is very small. On the other hand, when the stick is near the end of its stroke, a great stress is exerted on the elevator, the piston nears the bottom of its stroke, the deformation of the elastic system, under like stresses, is considerably reduced, and conditions are practically the same as if the control were rigid, especially at great speeds. With the control stick in a given position, which, without deformation of the elastic system, would correspond to an elevator angle $\beta_o$, the real deflection is $\beta$. As stated above, the aerodynamic hinge moment is

$$M'_c = \frac{n i_e + \rho\beta}{16} l' S V^2$$

This hinge moment is balanced by a force $F$ exerted by the stick

$$a F = M'_c$$

On the other hand $\alpha = a(\beta - \beta_o)$

Whence
$$F = \frac{4 \rho_o \omega d \alpha}{d^2 - 4\alpha^2}$$

We can write

$$M'_c = \frac{4 a^2 \rho_o \omega d (\beta - \beta_o)}{d^2 - 4a^2 (\beta - \beta_o)^2} = \frac{n i_e + \rho\beta}{16} l' S V^2$$

This relation enables us to calculate $\beta$. It appears in particular that

$$\text{for } V = 0, \qquad \beta = \beta_0,$$

$$\text{and for } V \to \infty, \quad ni_e + \rho\beta = 0 \quad \beta = -\frac{ni_e}{\rho}$$

The same conclusions would have been reached, if the calculation had been based on a different elastic system,- if still fulfilling the condition stated above. For a current value of $V$, $\beta$ lies between $\beta_0$ and $-ni_e/\rho$ and approaches the latter value as the speed increases. When $\beta = \beta_0$, the control is rigid. $\beta = -ni_e/\rho$ indicates that the elevator is free to turn about its hinge with control stick released. This leads to the following conclusions.

The elasticity of the control is small at low speed. The different positions of the control stick correspond to different elevator angles and hence to different incidences of equilibrium for the airplane which can fly in the zone $i_1 - i_2$. (Fig. 8.) When the speed increases, the zone $i_1\, i_2$ decreases $(i'_1 - i'_2)$, the points $i_1$ and $i_2$ approaching $i_0$. Finally, when the speed has become sufficiently large, the points $i_1$ and $i_2$ coincide with $i_0$, the incidence corresponding to the angle $\beta = -ni_e/\rho$.

This result enables us to affirm that such a control system reduces the stresses on the airplane structure, as shown by the following examples;

1. While flying at great speed, the pilot pulls the stick, in order to level off. The elevator deflection, limited as we have seen, guards the airplane against undue incidences and hence excessive lift, the aerodynamic stresses on the wing being thus limited.

2. The pilot pushes the control stick. The airplane assumes an incidence $i_1$ and begins a dive. The speed increases rapidly and the elasticity of the control begins to work. The elevator is depressed, $i_1$ shifts to the right, the airplane levels off, and the slope of the flight path is reduced, corresponding evidently to a lower airplane speed. At the maximum speed $V_M$, the slope of the flight path will correspond to an incidence $i'_1$, which corresponds itself to the speed $V_M$.

In Figure 9, curve 1 represents the speed variation in gliding flight plotted against the incidence. This speed is given by the expression

$$V^2 = \frac{T_1}{S\sqrt{K_y^2 + K_x^2}}$$

Curve 2 is the variation of the minimum incidence allowed by the elastic control (with stick pushed clear forward) plotted against the speed. The intersection of these two curves marks the speed limit not to be exceeded by the airplane. $V_M$ is smaller than $V_p$, the vertical diving speed. By thus limiting the speed, the stresses in leveling off will naturally be reduced, since they are proportional to the square of this speed.

## Stability of an Airplane with Elastic Controls

On the ground of the results obtained in the first part of this study, our stability investigation may be extended to airplanes provided with an elastic unit inserted between the control stick and the elevator. The elevator is subjected to the action of two moments, one aerodynamic and the other antagonistic, produced by the elastic unit (the stick being held).

The first moment tends to set the elevator at its angle of zero hinge moment, the other to coordinate its setting with the position of the stick, which would be the case if the control were rigid. The actual case is intermediate between that in which the stick is free and that in which it is held, the latter case being more closely approached as the speed decreases. This is confirmed by the following easy calculation.

Hinge moment due to aerodynamic loads.-

$$M'_c = \frac{Cm_c}{16} l' S V^2 = \frac{n i_e + \rho \beta}{16} l' S V^2 \tag{9}$$

$l'$: elevator chord = $\sigma l$

Hinge moment due to elastic system.- We will consider the simplest case, that of an elastic system reduced to a spring and a control stick in the neutral position. This permits a qualitative study of the general problem without extensive calculations.

$$M''_c = K\ \beta.$$

The resulting hinge moment is

$$M_c = \frac{ni_e + p\beta}{16}\ l'SV^2 + K\ \beta \tag{10}$$

$$M_c = 0 \qquad \beta = -\frac{ni_e\ l'SV^2}{pl'SV^2 + 16\ K} \tag{11}$$

By application of the same argument as for the case of the released stick, we obtain

$$F = \frac{Aie}{16}\left(1 - \frac{mn}{p + \frac{16K}{l'SV^2}}\right)SV^2 = \frac{Aie}{16}\ S'V^2 \tag{12}$$

whence

$$\boxed{\frac{S'}{S} = 1 - \frac{mn}{p + \frac{16\ K}{l'\ SV^2}}} \tag{13}$$

The value of the theoretical stabilizer area affording the same degree of stability is thus obtained. Obviously (13), for $V = 0$,

$$\frac{S'}{S} = 1$$

and for $V \nearrow \infty$

$$\frac{S'}{S} = 1 - \frac{mn}{p}$$

Hence the stability of airplanes with elastic controls can be determined by calculating, for each speed, the area $S'$ and plotting the corresponding stability curve against this area.

## II. STABILITY AND MANEUVERABILITY ABOUT THE LONGITUDINAL AXIS X

Static lateral stability does not exist. In the absence of all lateral and rotational velocity about the X axis, the same lift $f$ is exerted on both wings, the resultant $F$ always passing through the c.g. (Fig. 10.) If an angular velocity is imparted to the airplane, the incidence of the falling wing (2) increases by $\Delta i = \frac{v}{V}$ (fig.11) and that of the rising wing (i) decreases by i.

The resultant F no longer passes through the c.g. (Fig. 12.) It develops a stabilizing moment, if the incidence corresponds to the rising portion of the curve $(C_z, i)$, and a moment amplifying the motion, if $C_z$ maximum is exceeded (fig. 13) (autorotation). A lateral wind due to sideslipping has a similar effect. The leveling-off maneuver can be accelerated by the pilot and the autorotation damped, at least partially, by operating the ailerons. According to the incidence of the airplane, the magnitude of ω, the deflection and efficiency of the ailerons, one may obtain

$$C_{z_{2'}} > C_{z_{1'}}$$
$$\text{or } C_{z_{2''}} > C_{z_{1''}} \quad \text{(stability)}$$
$$\text{or } C_{z_{2'''}} < C_{z_{1'''}} \quad \text{(autorotation).}$$

Autorotation is always reduced by warping the wing.

Devices producing additional lift, such as leading-edge flaps connected with the ailerons, greatly improve lateral stability on account of the large intervals between the superposed $C_z$ curves of each wing and of the very large value of the critical angle.

In stating above that warping reduces autorotation, it was assumed that the deflection of an aileron introduces for each incidence a discrepancy $C_z$ variable with the incidence but of constant sign or, in other words, that increasing the incidence does not change the sign of the rolling moment (inversion of the aileron control).

This assumption is in contradiction of a commonly accepted opinion. It seems to be confirmed however, by the N.A.C.A. Reports on the experimental investigation of ailerons (Reference 1). In these documents the values of the rolling moment are measured for two different wings, provided with ailerons covering respectively 1/3, 1/2 and 2/3 of the semispan, the aileron chords varying from 15 to 35 per cent of the wing chord. The tests were made at angles varying from -44 to +44° and, in certain cases, with wing incidences reaching 48°. The sign of the rolling moment does not change in any case.

The diagram of the rolling moment plotted against the incidence for an aileron deflection of 20° is shown, for example, in Figure 14. This result was obtained with a Clark Y wing section. Similar results were obtained with the U.S.A. 27 and the N.A.C.A.-M 6 wing sections. All these sections are quite similar, and the above results may not be valid for sections with very different characteristics. This seems to justify the assumption that large ailerons are a good means for increasing the rolling moment and thus effectively opposing autorotation.

This solution has one drawback. The aileron being large, small motions imparted by the pilot to the control stick cause large lateral oscillations in rapid flight. This statement has already been made regarding the elevator control. The remedy then suggested may be applied in the present case. The ailerons should be controlled by a system with variable sensitivity, as described in connection with the elevator control. This would render it possible to maneuver the airplane at low speed, despite the large size of the ailerons.

As already mentioned, variable sensitivity may also be achieved by an elastic system of the type described above. The requisite condition is thus fulfilled and elasticity acquired as well. Such a system is shown in Figure 15. It is based on the use of an elastic post G which contracts as the speed increases. One important consequence of aileron deflection is the creation of a yawing moment due to the difference in the drag of the two wings. Banking is facilitated by this yawing moment, since the drag of the falling wing is stronger than that of the rising wing.

For a given incidence, the drag of conventional wings provided with standard ailerons is greater at positive (aileron depressed) than at negative angles. In this case the yawing moment is detrimental. Figure 16 shows the variation in $C_x$ for each wing $(C_{x_A} > C_{x_B})$. Two remedies are available.

1. The use of "Frieze"-type ailerons, the leading edge (a) of which (fig. 17) disturbs the air flow and increases the drag at negative angles.

2. Inequality of the deflections of the two ailerons, the rising one being deflected through a larger angle than the falling one. $\beta_1$ of one wing then corresponds to $-\beta'_1$, of the other wing $(\beta'_1 > \beta_1)$, as shown in Figure 16. Hence, banking is improved by the yawing moment, except for small deflections $(C_{x_A} < C_{x_C})$.

Conclusion.- Airplane flying characteristics, stability and maneuverability, are improved by special control systems. Safety and ease of piloting may be further increased, and researches of this order are desirable in each particular case.

Translation by W. L. Koperindé,
National Advisory Committee
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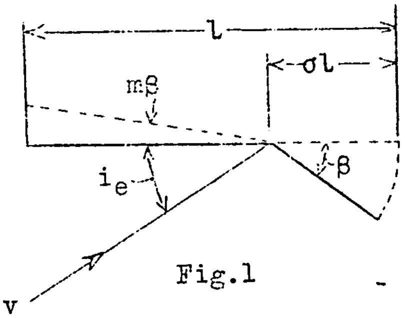



Fig.1

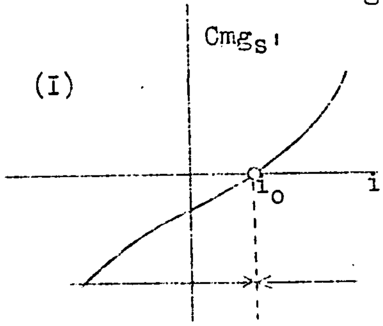



Fig.2

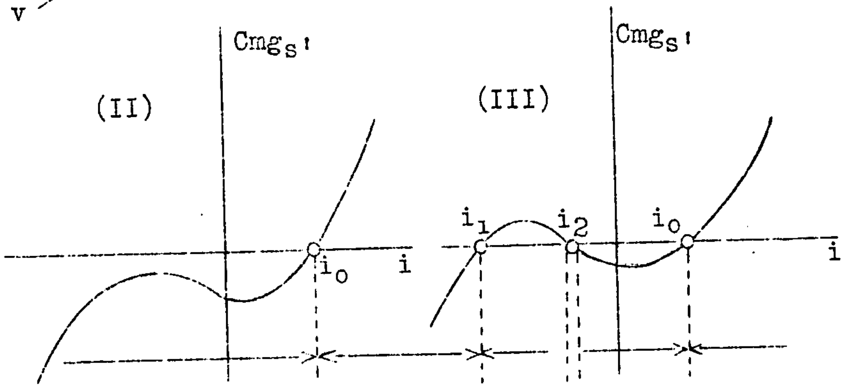



Fig.2

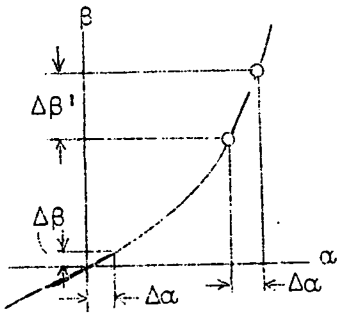



Fig.3

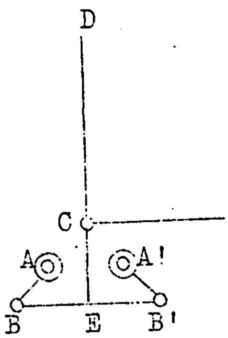



Fig.4

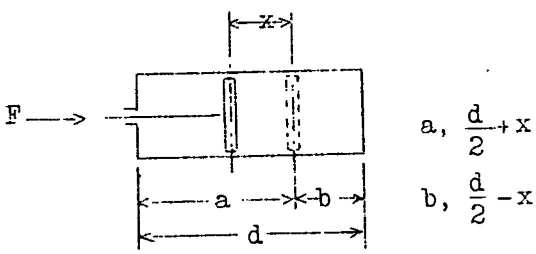



Fig.5

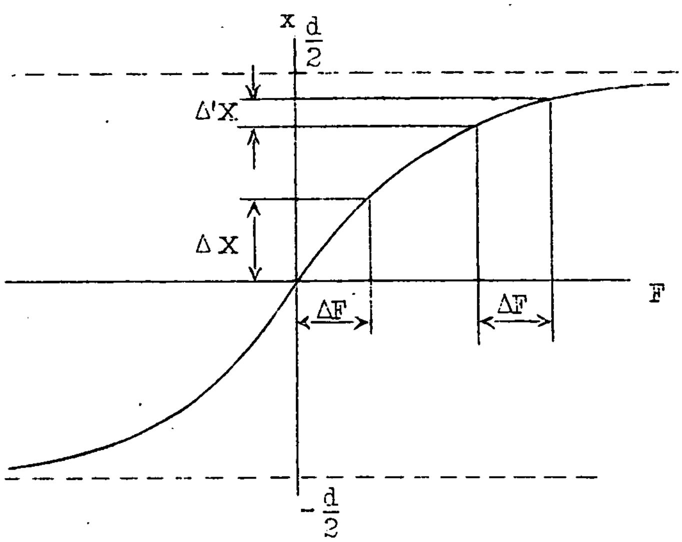



Fig. 6

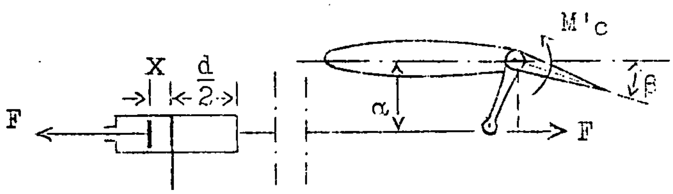



Fig. 7

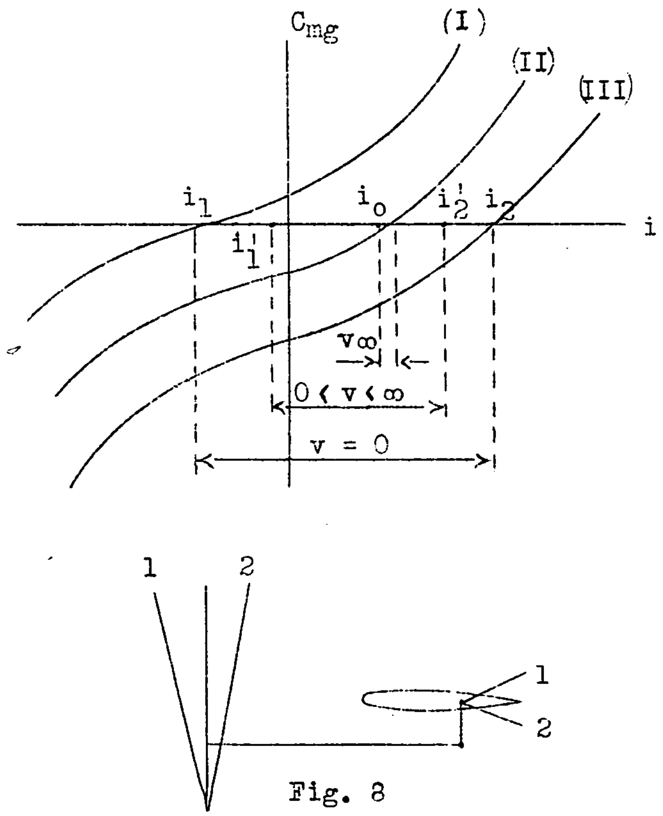



Fig. 8

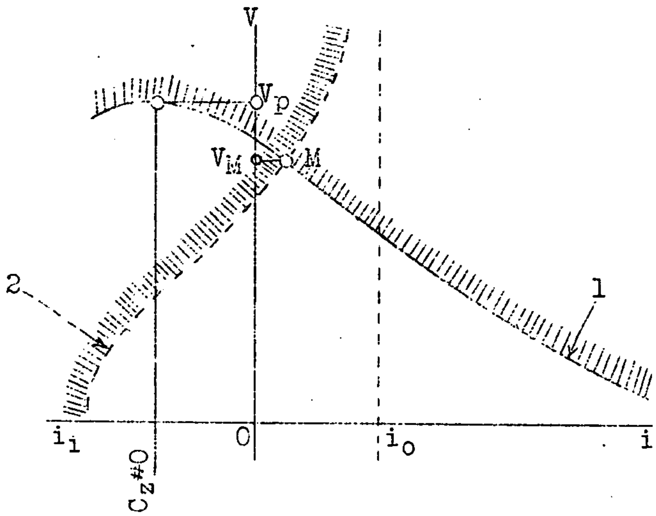



Fig.9

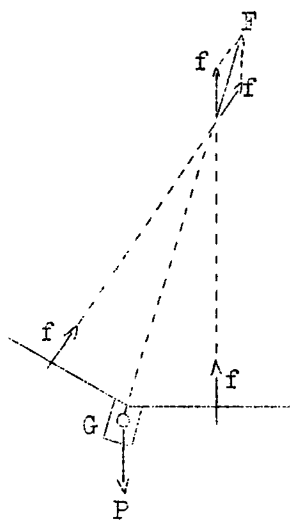



Fig.10

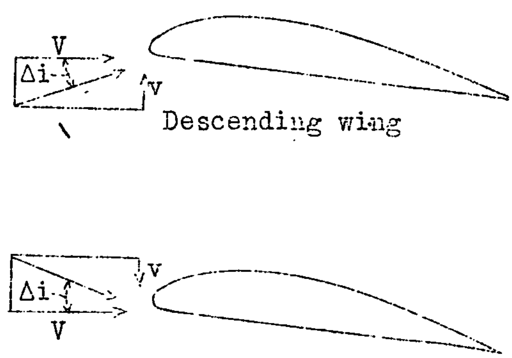



Fig.11

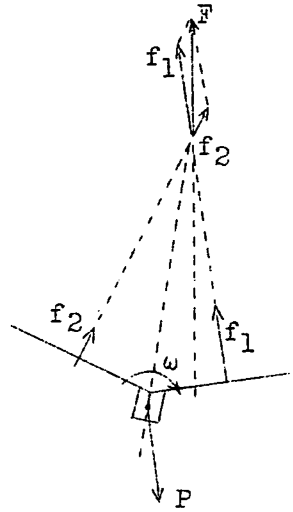
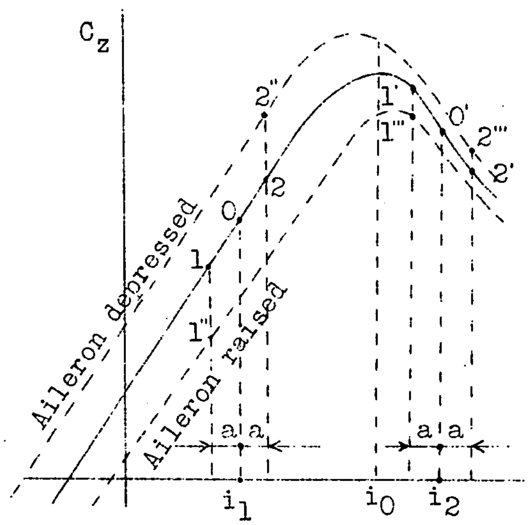



Fig.12




$a = \Delta i$

Fig.13

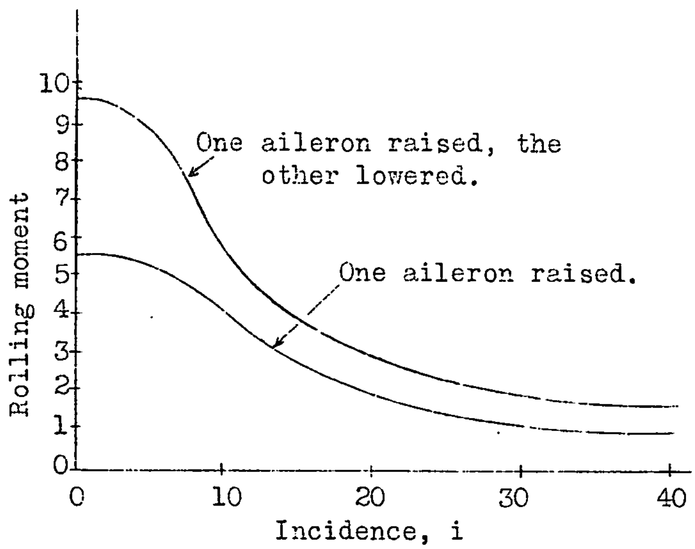



Fig.14

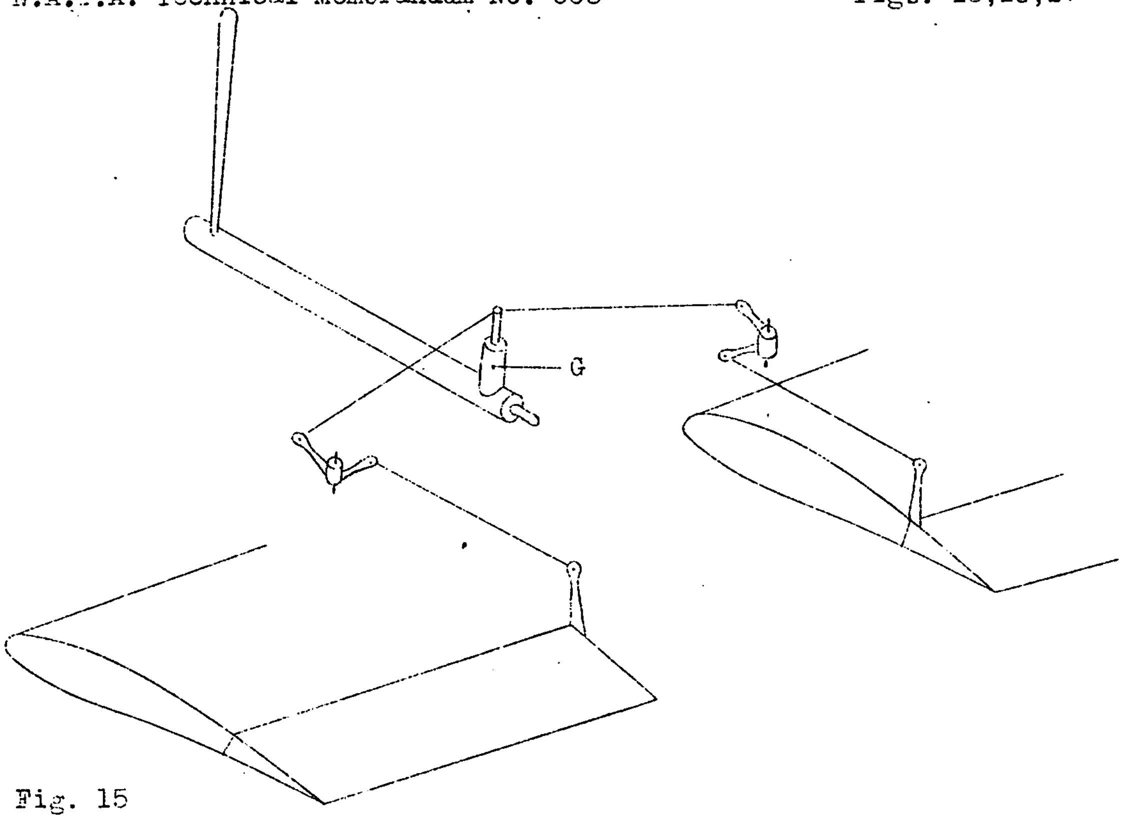



Fig. 15

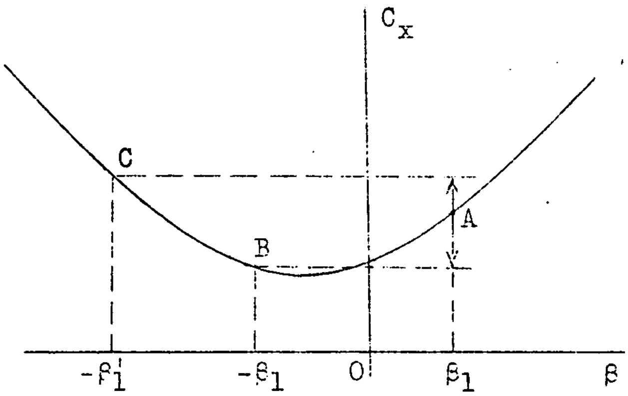



Fig. 16

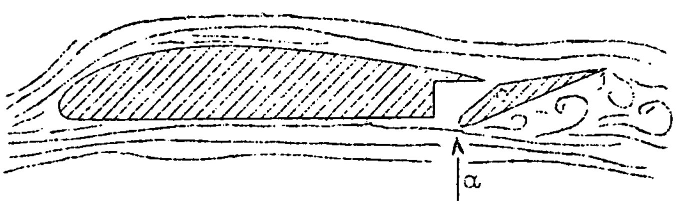



Fig. 17